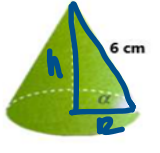


1



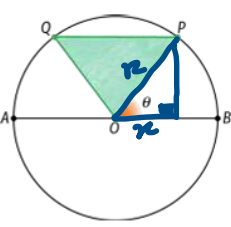
$$\alpha = 60^\circ$$
$$V = \frac{Ab \times h}{3}$$

$$\sin 60 = \frac{h}{6}$$
$$\Leftrightarrow h = 6 \sin 60$$
$$\Leftrightarrow h = 6 \times \frac{\sqrt{3}}{2}$$
$$\Leftrightarrow h = 3\sqrt{3} \quad (6)$$

$$= \frac{4 \times \frac{3^2 \times 3\sqrt{3}}{3}}{3}$$
$$= 9\sqrt{3} \text{ m}^3 \quad (8)$$

$$\cos 60 = \frac{R}{6}$$
$$\Leftrightarrow R = 6 \cos 60$$
$$\Leftrightarrow R = 6 \times \frac{1}{2}$$
$$\Leftrightarrow R = 3 \quad (6)$$

2



$$\overline{AB} = 8 \text{ cm}$$
$$R = 4$$
$$\theta = 52^\circ$$

$$\cos 52 = \frac{x}{4}$$
$$\Leftrightarrow x = 4 \cos 52 \longrightarrow \overline{OP} = 2 \times 4 \cos 52 = 8 \cos 52 \quad (4)$$
$$(8)$$

$$P = \frac{4 + 8 \cos 52 + 4}{2}$$
$$= \frac{8 + 8 \cos 52}{2}$$
$$= 12,9 \quad (8)$$

3

$$\sin x = \frac{5-3k}{2} \quad x \in \mathbb{R}$$

$$\text{Como } x \in \mathbb{R} \longrightarrow -1 \leq \sin x \leq 1 \quad (6)$$

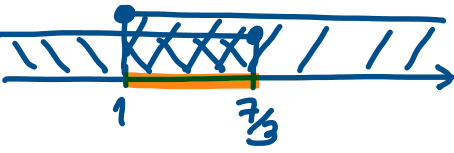
$$\Leftrightarrow -1 \leq \frac{5-3k}{2} \leq 1 \quad (6)$$

$$\Leftrightarrow -1 \leq \frac{5-3k}{2} \wedge \frac{5-3k}{2} \leq 1$$

$$\Leftrightarrow -2 \leq 5-3k \wedge 5-3k \leq 2$$

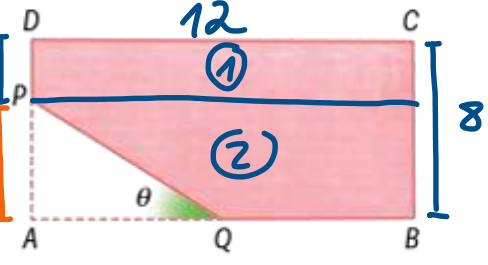
$$\Leftrightarrow 3k \leq 7 \wedge -3k \leq -3$$

$$\Leftrightarrow k \leq \frac{7}{3} \wedge k \geq 1 \quad (6)$$



$$k \in \left[1, \frac{7}{3}\right] \quad (2)$$

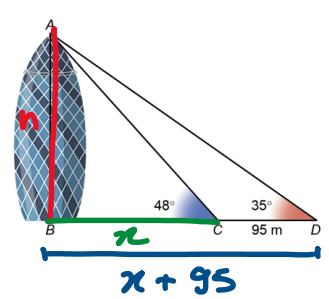
4



$$\theta = 30^\circ$$
$$\tan 30 = \frac{6}{AQ}$$
$$\Leftrightarrow \frac{\sqrt{3}}{3} = \frac{6}{AQ}$$
$$\Leftrightarrow AQ = \frac{18}{\sqrt{3}}$$
$$\Leftrightarrow AQ = 6\sqrt{3}$$
$$QB = 12 - 6\sqrt{3} \quad (4)$$

$$A = A_1 + A_2$$
$$= 12 \times 2 + \frac{12 + 12 - 6\sqrt{3}}{2} \times 6$$
$$= 24 + (12 - 3\sqrt{3}) \times 6$$
$$= 24 + 72 - 18\sqrt{3}$$
$$= 96 - 18\sqrt{3} \quad (4)$$

5

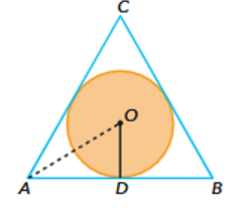


$$\tan 40 = \frac{h}{x} \Leftrightarrow h = x + 95 \tan 40$$
$$\tan 30 = \frac{h}{x+95} \Leftrightarrow h = (x+95) \tan 30$$
$$\Leftrightarrow x + 95 \tan 40 = x \tan 30 + 95 \tan 30$$

$$\begin{cases} \text{---} \\ x(\tan 40 - \tan 30) = 95 \tan 30 \end{cases} \Leftrightarrow \begin{cases} h = \frac{95 \tan 30}{\tan 40 - \tan 30} \times \tan 40 \\ x = \frac{95 \tan 30}{\tan 40 - \tan 30} \end{cases} \Leftrightarrow \begin{cases} h \approx 180,0 \\ x \approx 162,1 \end{cases} \quad (24,1)$$

A altura do edifício é 180,0 m

6



Como o triângulo é equilátero $\rightarrow \widehat{BAC} = 60^\circ$ (4)

Seja O o incentro (centro da circunferência inscrita), AO é bissetriz do ângulo BAC, logo:

$$\widehat{BAO} = \frac{60}{2} = 30^\circ \quad (6)$$

$$AO = \frac{4\sqrt{2}}{2} = 2\sqrt{3} \quad (4)$$


$$\tan 30 = \frac{R}{2\sqrt{3}} \quad (6)$$
$$\Leftrightarrow R = 2\sqrt{3} \tan 30$$
$$\Leftrightarrow R = 2 \quad (4)$$
$$A_0 = \pi \times 2^2 = 4\pi \quad (6)$$

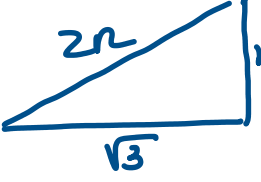
ou

Como o triângulo é equilátero as pontos notáveis coincidem: (6)

$\rightarrow$ O incentro coincide com o baricentro, logo $\overline{CO} = \frac{2}{3} \overline{OB}$

$\rightarrow$ O incentro coincide com o circuncentro, logo $\overline{AO} = \overline{CO}$ (4)

Assim: $\overline{AO} = \frac{2}{3} \overline{OB}$



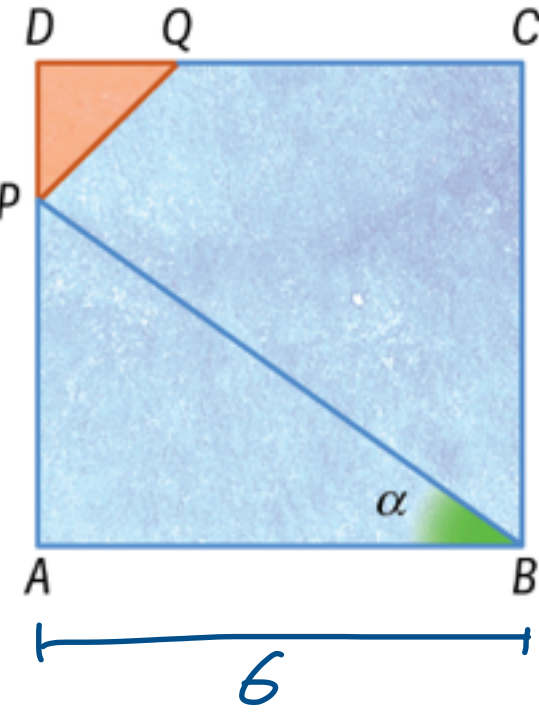
Reco T.P.

$$(2R)^2 = R^2 + (2\sqrt{3})^2 \quad (6)$$
$$\Leftrightarrow 4R^2 = R^2 + 12$$
$$\Leftrightarrow 3R^2 = 12$$
$$\Leftrightarrow R^2 = 4 \quad (4)$$

Como R é um comprimento

$$R = \sqrt{4} = 2 \longrightarrow A = \pi \cdot 2^2 = 4\pi \quad (6)$$

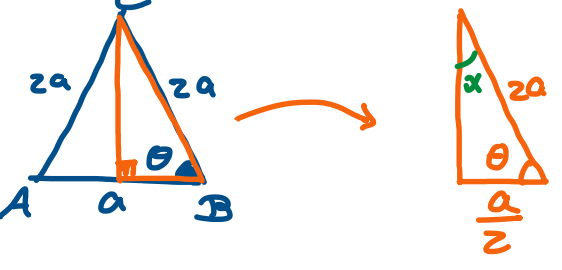
7



$$\tan \alpha = \frac{AP}{6}$$
$$\Leftrightarrow AP = 6 \tan \alpha$$
$$DP = 6 - 6 \tan \alpha \quad (4)$$

$$A_\Delta = \frac{b \times h}{2}$$
$$= \frac{(6 - 6 \tan \alpha)(6 - 6 \tan \alpha)}{2} \quad (10)$$
$$= \frac{(6 - 6 \tan \alpha)^2}{2}$$
$$= \frac{[6(1 - \tan \alpha)]^2}{2}$$
$$= \frac{36(1 - \tan \alpha)^2}{2}$$
$$= 18(1 - \tan \alpha)^2 \quad (10)$$

8



$$\cos \theta = \frac{\frac{a}{2}}{\frac{2a}{1}} = \frac{a}{4a} = \frac{1}{4} \longrightarrow \text{logo a proposição 1 é falsa.} \quad (10)$$
$$\cos \theta = \frac{1}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{4}\right)$$
$$\alpha = 30 - \cos^{-1}\left(\frac{1}{4}\right)$$
$$\widehat{ACB} = 2 \times \left(30 - \cos^{-1}\left(\frac{1}{4}\right)\right) \neq 30^\circ \quad ([ABC] \text{ é um triângulo isósceles})$$

↓

logo a proposição 2, também é falsa (10)