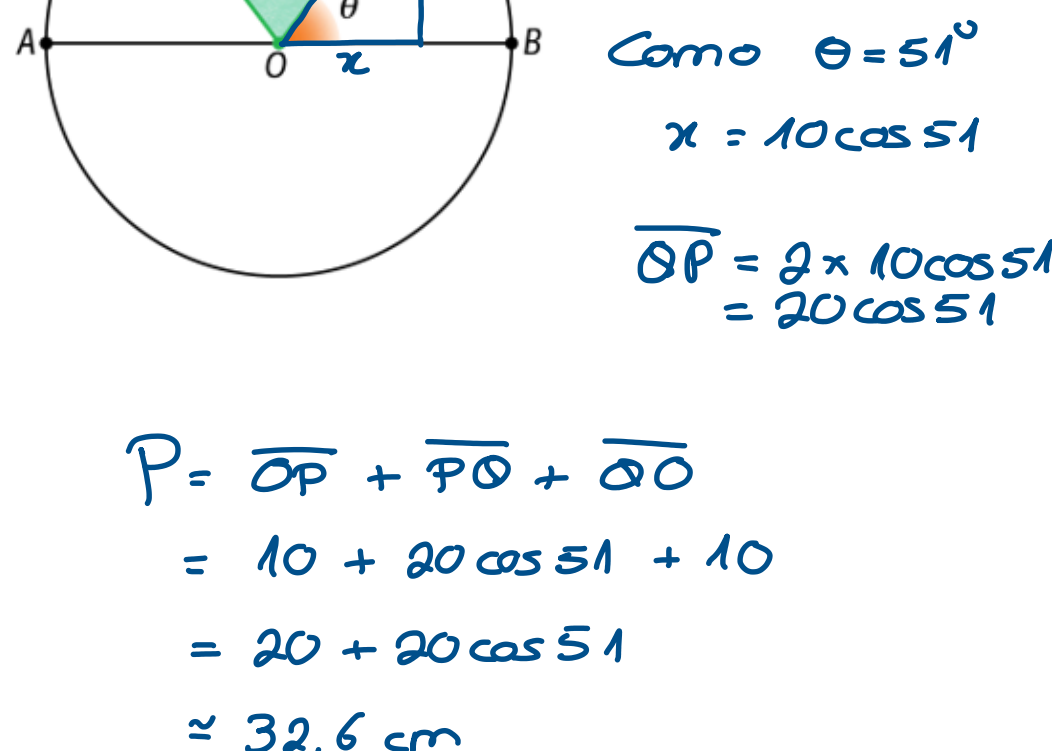


1

$$\begin{aligned} \sin \beta &= \frac{\overline{CD}}{3} \\ \Leftrightarrow \overline{CD} &= 3 \sin \beta \end{aligned} \quad \left\{ \begin{aligned} \sin 60 &= \frac{\overline{CD}}{\overline{AC}} \\ \Leftrightarrow \frac{\sqrt{3}}{2} &= \frac{3 \sin \beta}{\overline{AC}} \\ \Leftrightarrow \overline{AC} &= \frac{6 \sin \beta}{\sqrt{3}} \\ \Leftrightarrow \overline{AC} &= \frac{6 \sqrt{3} \sin \beta}{3} \\ \Leftrightarrow \overline{AC} &= 2 \sqrt{3} \sin \beta \end{aligned} \right.$$

→ opção B

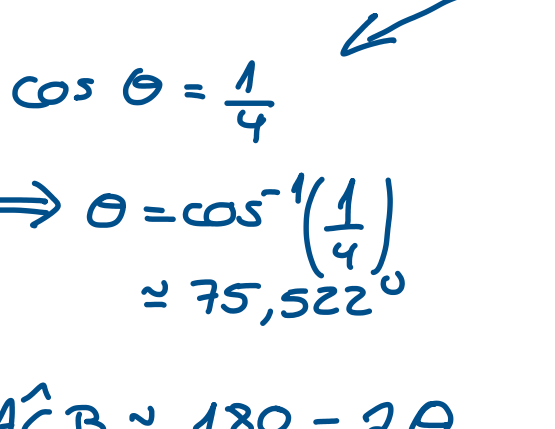
2



$$\begin{aligned} \cos \theta &= \frac{x}{10} \\ \Leftrightarrow x &= 10 \cos \theta \\ \text{Como } \theta &= 51^\circ \\ x &= 10 \cos 51 \\ \overline{OP} &= 2 \times 10 \cos 51 \\ &= 20 \cos 51 \end{aligned}$$

$$\begin{aligned} P &= \overline{OP} + \overline{PO} + \overline{AO} \\ &= 10 + 20 \cos 51 + 10 \\ &= 20 + 20 \cos 51 \\ &\approx \underline{\underline{32,6 \text{ cm}}} \end{aligned}$$

3



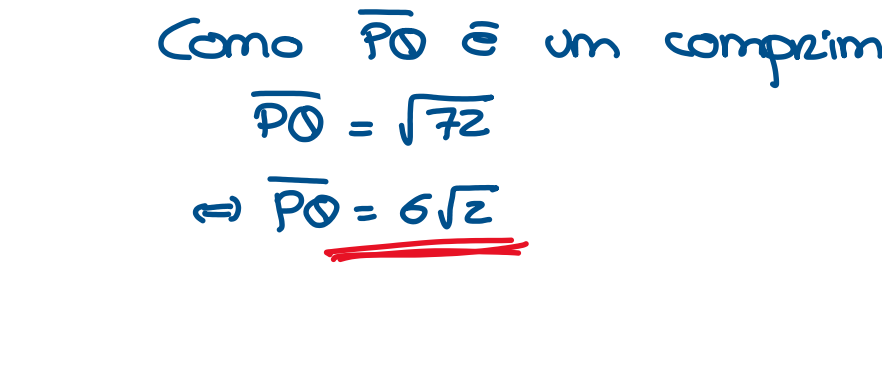
$$\begin{aligned} \cos \theta &= \frac{20}{20} \\ &= \frac{1}{1} \\ &= \frac{1}{1} \end{aligned}$$

Logo a afirmação I é falsa

$$\begin{aligned} \cos \theta &= \frac{1}{4} \\ \Rightarrow \theta &= \cos^{-1}\left(\frac{1}{4}\right) \\ &\approx 75,522^\circ \end{aligned}$$

$$\begin{aligned} \widehat{ACB} &\approx 180 - 2\theta \\ &= 180 - 2 \times 75,522 \\ &= 28,955 \neq 60^\circ \rightarrow \text{Logo a afirmação II também é falsa.} \end{aligned}$$

4



$$\overline{PQ}^2 = 6^2 + 6^2$$

$$\begin{aligned} \Leftrightarrow \overline{PQ}^2 &= 36 + 36 \\ \Leftrightarrow \overline{PQ}^2 &= 72 \\ \text{Como } \overline{PQ} &\text{ é um comprimento} \\ \overline{PQ} &= \sqrt{72} \\ \Leftrightarrow \overline{PQ} &= \underline{\underline{6\sqrt{2}}} \end{aligned}$$

$$\begin{aligned} P &= 6 + 8 + 12 + 2 + 6\sqrt{2} \\ &\approx \underline{\underline{28 + 6\sqrt{2}}} \\ &\approx \underline{\underline{36}} \end{aligned}$$

$$8 - 2 = 6$$

$$\theta = 45^\circ$$

$$\tan 45 = \frac{6}{x}$$

$$\Leftrightarrow 1 = \frac{6}{x}$$

$$\Leftrightarrow x = \frac{6}{1}$$

$$\Leftrightarrow x = 6 \rightarrow \overline{QB} = 12 - 6 = 6$$

b)

$$\theta = 30^\circ$$



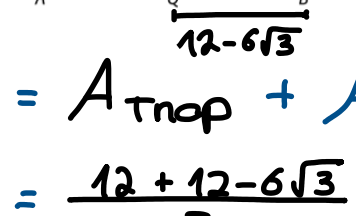
$$\tan 30 = \frac{6}{\overline{AB}}$$

$$\frac{\sqrt{3}}{3} = \frac{6}{\overline{AB}}$$

$$\overline{AB} = \frac{6 \times 3 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\overline{AB} = 6\sqrt{3}$$

$$\overline{QB} = 12 - 6\sqrt{3}$$



$$A = A_{\text{trap}} + A_{\text{rect}}$$

$$= \frac{12 + 12 - 6\sqrt{3}}{2} \times 6 + 12 \times 2$$

$$= (12 - 3\sqrt{3}) \times 6 + 24$$

$$= 72 - 18\sqrt{3} + 24$$

$$= \underline{\underline{96 - 18\sqrt{3}}}$$

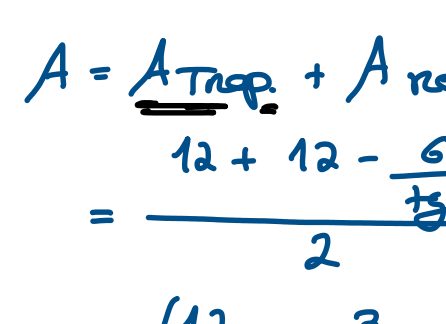
c)



$$\tan \theta = \frac{6}{\overline{AB}}$$

$$\overline{AB} = \frac{6}{\tan \theta}$$

$$\overline{QB} = 12 - \frac{6}{\tan \theta}$$



$$A = A_{\text{trap}} + A_{\text{rect}}$$

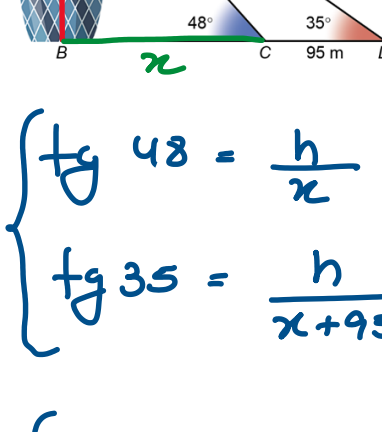
$$= \frac{12 + 12 - \frac{6}{\tan \theta}}{2} \times 6 + 12 \times 2$$

$$= \left(12 - \frac{3}{\tan \theta}\right) \times 6 + 24$$

$$= 72 - \frac{18}{\tan \theta} + 24$$

$$= \underline{\underline{96 - \frac{18}{\tan \theta}}}$$

5

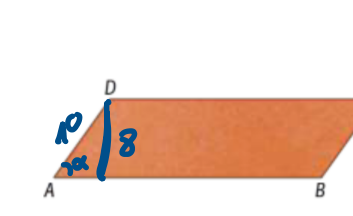


$$\begin{cases} \tan 48 = \frac{h}{x} \\ \tan 35 = \frac{h}{x+95} \end{cases} \Rightarrow \begin{cases} h = x \tan 48 \\ h = (x+95) \tan 35 \end{cases} \Rightarrow \begin{cases} x \tan 48 = x \tan 35 + 95 \tan 35 \\ x \tan 48 - x \tan 35 = 95 \tan 35 \end{cases}$$

$$\begin{cases} x (\tan 48 - \tan 35) = 95 \tan 35 \\ \Leftrightarrow x = \frac{95 \tan 35}{\tan 48 - \tan 35} \end{cases} \Rightarrow \begin{cases} h = \frac{95 \tan 35}{\tan 48 - \tan 35} \times \tan 48 \\ h \approx 180,012 \\ x \approx 162,083 \end{cases}$$

$$\underline{\underline{h \approx 180 \text{ m}}}$$

6



$$A = 240$$

a)

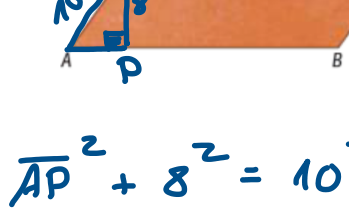
$$A = 240$$

$$\Leftrightarrow b \times h = 240$$

$$\Leftrightarrow 30 \times h = 240$$

$$\Leftrightarrow \underline{\underline{h = 8 \text{ cm}}}$$

b)



$$\sin \alpha = \frac{8}{10} \Rightarrow \alpha = \sin^{-1}\left(\frac{8}{10}\right) = 53,1^\circ$$

$$\widehat{BAP} = \widehat{DCB} = 53,1^\circ$$

$$\widehat{ADC} = \widehat{ABC} = 26,9^\circ$$

c)



$$\overline{AP}^2 + 8^2 = 10^2$$

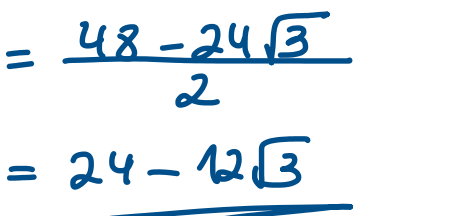
$$\Leftrightarrow \overline{AP}^2 = 100 - 64$$

$$\Leftrightarrow \overline{AP}^2 = 36$$

$$\text{Como } \overline{AP} \text{ é um comprimento}$$

$$\underline{\underline{\overline{AP} = 6}}$$

7



$$\overline{AB} = 6 \text{ cm}$$

$$\alpha = 30^\circ$$

$$\tan 30 = \frac{\overline{AP}}{\overline{AB}}$$

$$\Leftrightarrow \frac{\sqrt{3}}{3} = \frac{\overline{AP}}{6}$$

$$\Leftrightarrow \overline{AP} = \frac{6\sqrt{3}}{3}$$

$$\Leftrightarrow \overline{AP} = \underline{\underline{2\sqrt{3}}}$$

$$\overline{DB} = \overline{DP} = 6 - 2\sqrt{3}$$

$$A_{\Delta} = \frac{b \times h}{2}$$

$$= \frac{(6 - 2\sqrt{3})(6 - 2\sqrt{3})}{2}$$

$$= \frac{36 - 12\sqrt{3} - 12\sqrt{3} + 4 \times 3}{2}$$

$$= \frac{48 - 24\sqrt{3}}{2}$$

$$= \underline{\underline{24 - 12\sqrt{3}}}$$

b)

$$A_{\Delta} = 10$$

$$\Leftrightarrow \frac{b \times h}{2} = 10$$

$$\Leftrightarrow \frac{x \times x}{2} = 10$$

$$\Leftrightarrow x^2 = 20$$

$$\text{Como } x \text{ é um comprimento}$$

$$x = \sqrt{20}$$

$$\overline{AP} = 6 - \sqrt{20}$$

$$\tan \alpha = \frac{6 - \sqrt{20}}{6}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{6 - \sqrt{20}}{6}\right)$$

$$= 14,3^\circ$$

c)

$$\tan \alpha = \frac{\overline{AP}}{6}$$

$$\overline{AP} = 6 \tan \alpha \Rightarrow \overline{PD} = 6 - 6 \tan \alpha$$

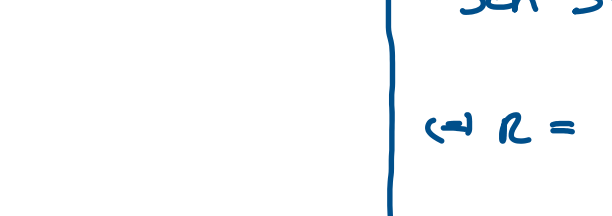
$$A_{\Delta} = \frac{(6 - 6 \tan \alpha)^2}{2}$$

$$= \frac{(6(1 - \tan \alpha))^2}{2}$$

$$= \frac{36(1 - \tan \alpha)^2}{2}$$

$$= 18(1 - \tan \alpha)^2$$

8



$$\sin 10 = \frac{\overline{BC}}{2}$$

$$\Leftrightarrow \overline{BC} = 2 \sin 10$$

$$\Leftrightarrow \overline{BC} \approx 0,34 \text{ m}$$

$$\underline{\underline{= 34 \text{ cm}}}$$

9



$$\overline{AC} = 2 \text{ cm}$$

$$\overline{BD} = 6 \text{ cm}$$

Ángulo encajado

a)

$$\tan \alpha = \frac{1}{3}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{3}\right) = 18,435$$

$$\beta = 2 \times \alpha$$

$$= 2 \times 18,435$$

$$= \underline{\underline{36,87^\circ}}$$

b)



$$\begin{aligned} \sin 30 &= \frac{\overline{DO}}{6} \\ \Leftrightarrow \overline{DO} &= 6 \sin 30 \\ \cos 30 &= \frac{\overline{AO}}{6} \\ \Leftrightarrow \overline{AO} &= 6 \cos 30 \\ \Leftrightarrow \overline{AO} &= 3\sqrt{3} \end{aligned}$$

$$\overline{AC} = 2 \times 3 = 6 \text{ cm}$$

$$\overline{BD} = 2 \times 3\sqrt{3} = 6\sqrt{3} \text{ cm}$$

c) Se as diagonais tiverem o mesmo comprimento, o losango é um quadrado.

$$\underline{\underline{\angle \beta = 90^\circ}}$$

10

reta tangente - perpendicular ao raio no ponto de tangência

$$\tan 54 = \frac{r}{10}$$

$$\Leftrightarrow 10 = \frac{r}{\tan 54}$$

$$\sin 54 = \frac{r}{10}$$

$$\Leftrightarrow r = \frac{10}{\tan 54} \times \sin 54$$

$$\Leftrightarrow r = \frac{10 \cos 54}{\sin 54} \times \sin 54$$

$$\Leftrightarrow r = 2 \cos 54$$

$$\Leftrightarrow \underline{\underline{r = 1,2 \text{ cm}}}$$

11

Como o círculo está inscrito no triângulo, O é incentro. Assim, O é bissetriz do ângulo BAC.

$$\widehat{BAC} = 60^\circ \text{ (triângulo equilátero)}$$

$$\widehat{BAO} = \frac{60}{2} = 30^\circ$$

$$\tan 30 = \frac{r}{\sqrt{3}}$$

$$\Leftrightarrow r = \sqrt{3} \tan 30$$

$$\Leftrightarrow r = \sqrt{3} \times \frac{\sqrt{3}}{3}$$

$$\Leftrightarrow \underline{\underline{r = 1}}$$

$$AO = \frac{10 \times r}{2}$$

$$= \frac{10 \times 1}{2}$$

$$= \underline{\underline{5}}$$

Como o triângulo é equilátero, os 4 pontos notáveis coincidem. Assim, o incentro é também baricentro dividindo, desta forma, [CD] na razão de 2 para 1:

$$\overline{CO} = 2 \overline{OD}$$

$$\text{Como } \overline{CO} = \overline{AO}$$

$$\overline{AO} = 2 \overline{OD}$$

$$\overline{AO} = 2 \overline{OD}$$

$$(2r)^2 = r^2 + \sqrt{3}^2$$

$$\Leftrightarrow 4r^2 = r^2 + 3$$

$$\Leftrightarrow 3r^2 = 3$$

$$\Leftrightarrow r^2 = 1$$

$$\text{Como } r \text{ é um comprimento}$$

$$r = \sqrt{1} = \underline{\underline{1}}$$

$$A = \frac{\sqrt{3} r^2}{4}$$

$$= \frac{\sqrt{3} \times 1^2}{4}$$

$$= \underline{\underline{\frac{\sqrt{3}}{4}}}$$